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DETERMINATION OF THE STRESSES PRODUCED BY THE LANDING IMPACT IN THE BULKHEADS OF A SEAPLANE BOTTOM

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DETERMINATION OF THE STRESSES PRODUCED BY THE LANDING IMPACT IN THE BULKHEADS OF A SEAPLANE BOTTOM*

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INTRODUCTION

The present report deals with the determination of the impact stresses in the bulkhead floors of a seaplane bottom. The dynamic problem is solved on the assumption of a certain elastic system, the floor being assumed as a weightless elastic beam with concentrated masses at the ends (due to the mass of the float) and with a spring which replaces the elastic action of the keel in the center. The distributed load on the floor is that due to the hydrodynamic force acting over a certain portion of the bottom. The pressure distribution over the width of the float is assumed to follow the Wagner law. The formulas given for the maximum bending moment are derived on the assumption that the keel is relatively elastic, in which case it can be shown that at each instant of time the maximum bending moment is at the point of juncture of the floor with the keel. The bending moment at this point is a function of the half width of the wetted surface c and reaches its maximum value when c is approximately equal to $b/2$ where b is the half width of the float. In general, however, for computing the bending moment the values of the bending moment at the keel for certain values of c are determined and a curve is drawn. The illustrative sample computation gave for the stresses a result approximately equal to that obtained by the conventional factory computation.

METHOD OF COMPUTATION

Of the possible cases of the landing of a seaplane the most critical case will be that for which the hydrodynamic impact is most nearly central (landing on the step).

*Report No. 449, of the Central Aero-Hydrodynamical Institute, Moscow, 1939.

This case, therefore, will be the basis of this discussion. In the cases of landing on the bow or stern the length of the impact surface is sufficient for the impact phenomenon to be schematized as that of an infinitely long wedge on water. (The problem has been solved by Wagner (reference 1).) In the case of landing on the step, if the landing is effected on a calm surface, the length of the impact surface is small, and account must be taken of the fact that the float enters the water at a certain angle. In the most severe case, however - namely, landing on a large wave - the length of the impact surface exceeds the width by at least 1.5 times. Hence, for this case the preceding schematization is permissible.

In landing on the step when the ratio of the length of the impact surface to the width is near 1.5, a correction for finiteness of the surface is nevertheless desirable. This correction may be made with the aid of the experimental formulas obtained by Pabst (reference 2) and Povitsky (reference 3).

The number of main bulkheads associated with the impact surface is not large. These bulkheads generally differ little from each other. For simplicity and with small error they will be assumed as working under the same conditions. In accordance with the preceding assumption consider that the bottom of each of the bulkheads associated with the impact surface is loaded by a force P/n where P is the total impact force and n the number of bulkheads considered. In the case where certain of the bulkheads differ appreciably in stiffness it is sometimes possible to assume that the loading on the bulkhead bottoms is proportional to their stiffnesses. Since the bottom always has considerably less stiffness than the remaining structure (hull) of the float, consider the latter as absolutely rigid and only the bottom as elastic*. In this case in the landing on water the inertia forces will be transmitted mainly at the ends of the bulkhead floors. This permits a consideration of the mass of the entire float as

*A misunderstanding may arise. On the one hand use is made of the Wagner solution which is applicable to a wedge of absolute stiffness and, on the other hand, the elasticity of the bottom. The fact is that the hydrodynamic pressure is determined by the Wagner method. For a V-bottom float, however, the change in pressure for small changes in the bottom shape due to elasticity is insignificant. This change should be taken into account only for practically flat bottoms.

concentrated at the ends of the floors*. If the mass of the entire float is denoted by m, the mass associated with each end of the floor is m/2n.

In the case of eccentric impact, instead of m, $m_r = m\frac{i^2}{i^2 + r^2}$ should be used where i is the radius of inertia of the seaplane and r the lever arm relative to the center of gravity of the seaplane.

The centers of the floor beams will be acted on by the elastic force of the bottom with the force transmitted mainly through the keel. This force arises from the deformation of the bottom on impact. The elastic force of the bottom is considered as the elastic force of a fixed keel beam supported on intermediate elastic supports (the floors lying beyond the impact surface).

In order to take into account, to some extent, the action of the skin, the keel beam and the bulkhead floors, with the attached strips of skin to be of a width of 40 times the thickness, are considered together.

Since the local action of the impact extends over a finite length of bottom, it will be sufficient to consider the keel beam to be fixed on three main bulkheads to the left and the right of the impact surface. Thus the part of the keel of interest may be represented by a built-in beam on four intermediate elastic supports (fig. 1). The rigidity of these supports κ is determined by the formula $\kappa = \frac{6EI_\Phi}{b^3}$ where 2b is the length of the floor, EI_Φ the stiffness (the floors are considered as straight beams of constant stiffness).

The method of computing beams on elastic supports is described in detail in the work of Umansky (reference 4). By use of this method the maximum deflection due to the concentrated force at the center of the beam P is determined

$$f = \frac{P}{K}$$

*The bulkhead floors associated with the impact surface only are considered.

where

$K = K(EI_K, \kappa, \Delta l)$

EI_K stiffness of the keel beam

κ stiffness of the supports (the ratio of the reaction of the supports to their deflection)

Δl spacing between supports

The elastic action exerted on the bulkhead floors of the wetted surface by the rest of the bottom structure is thus replaced by the action of an equivalent spring of stiffness K. Each of the floors is acted on by a spring of stiffness K/n.

As a result the following scheme is arrived at. The elastic bent beam (floor) with the concentrated masses $m/2n$ at the ends (due to the mass of the float) has at the instant $t = 0$ a velocity V_o (initial velocity of immersion of float*). Thereafter the beam is acted upon by the distributed load P/n (hydrodynamic pressure) and the concentrated forces (elastic action of the bottom). At the center of the beam the force $F = \frac{KS}{n}$ where S is the vertical displacement of the keel with respect to the chine, and at the ends of the beam the forces are $F/2$ (fig. 2)**.

On account of the symmetry one-half the beam may be considered, the corresponding boundary conditions being applied to the second half and the spring force taken to be $F/2$ - that is, the stiffness decreased by one-half.

The floors may be of the most varied shapes. Two sharply differing shapes are shown in figures 3 and 4. The elastic axis of the floor is, in general, curved. However, it shall be considered straight, since the error made in this assumption should be insignificant. The elastic axis may be drawn as indicated in figures 3 and 4. Furthermore, the stiffness of the floor shall

*For landing on the step V_o should be taken as the component of the landing velocity normal to the keel at the step. In the remaining cases V_o may be taken as the vertical component of the landing velocity.

**The dotted line represents the stiff frame replacing the float hull.

be considered as constant (mean stiffness) although a variable stiffness offers no fundamental difficulties for the discussion. It may be noted that for modern floats there is a tendency to make the floor beam of almost constant height. In view of the fact that the elastic axis is inclined at a certain angle to the surface of the water and the pressure of the latter is not normal to the axis, the floor beam during the landing of the seaplane experiences both a transverse and a longitudinal impact. By decomposing the forces acting on the floor beam, including the inertia forces, into two components (along and perpendicular to the elastic axis) the transverse and longitudinal impacts may be separately considered, the stress in the floor beam being determined as the sum of the bending and compressive stresses. It is easy to show, however, by simple computation that the compressive stresses will be negligibly small by comparison with the bending stresses and hence the longitudinal impact need not be considered. In exactly the same way, it is of no value to take into account the inclination of the elastic axis to the water surface, since this angle α will generally be 10° to 20°, $\cos \alpha = 0.98430 - 0.93869$, and hence the corresponding components of the forces will differ little from the values of the latter themselves.

Thus the final scheme (figs. 5 and 6) will be somewhat simpler than that shown in figure 2. Figure 5 shows the scheme at the initial instant $t = 0$, figure 6 at the instant t when the float is immersed somewhat in the water. As in figure 2, the dotted line represents the stiff frame replacing the upper rigid part of the float. Since the mass of the beam (floor) shown in figure 5 is small by comparison with the mass concentrated at its right end, the beam is considered as weightless.

The load distribution on the beam is determined with the aid of the formula of Wagner (reference 5) for the hydrodynamic pressure on a submerged wedge:

$$p(x,c) = \frac{V_o^2}{(1+\mu)^2}\,\frac{\rho}{u}\left[\frac{1+u^2}{\sqrt{1+u^2-\frac{x^2}{c^2}}} - \frac{2\mu}{1+\mu}\sqrt{1+u^2-\frac{x^2}{c^2}} + \frac{cu}{\sqrt{1+u^2-\frac{x^2}{c^2}}}\,\frac{du}{dc} - \frac{1}{2}\,\frac{u}{\frac{c^2}{x^2}(1+u^2)-1}\right] \tag{1}$$

where

ρ density of water

$\mu = \frac{\rho \pi c^2 l}{2m}$

c half width of wetted surface of bottom

l length of wetted surface

and u is determined by the formula

$$u = \frac{2}{\pi}\left\{\tan\beta + c\int_0^c \frac{d}{d\xi}\left[\frac{\eta(\xi)}{\xi}\right]\frac{d\xi}{\sqrt{c^2-\xi^2}}\right\} \tag{2}$$

β V angle

$\eta = \eta(\xi)$ equation of bottom profile described relative to the axes of coordinates shown in figure 7

If the bottom profile $\eta(\xi)$ may be expressed in the form of the series*

$$\eta(\xi) = \beta_0\xi + \beta_1\xi^2 + \beta_2\xi^3 + \ldots$$

then $u(c)$ is very simply determined as

$$u(c) = k_0\beta_0 + k_1\beta_1 c + k_2\beta_2 c^2 + \ldots$$

where

$k_0 = 0.636$, $k_1 = 1.0$, $k_2 = 1.272$,
$k_3 = 1.5$, $k_4 = 1.696$, $k_5 = 1.875$

and so forth**

The linear loading on the beam therefore will be

$$q(x,c) = \begin{cases} -p(x,c)\ \Delta l & \text{at } [0,c] \\ 0 & \text{at } [c,b] \end{cases}$$

where

$\Delta l = \frac{l}{n}$

l length of impact surface

b length of beam (half width of float)

*Generally, for surfaces without discontinuities, the profile is sufficiently well represented by an algebraic curve of the form

$$\eta = \beta_0\xi - \beta_{n-1}\xi^n$$

**The general formula for k_n may be found in the Airplane Constructor's Reference Book, vol. II, 1938.

The equation of motion of the points of the elastic axis of the beam with linear loading $q(x,c)$ is the following

$$EI\frac{\partial^4 y}{\partial x^4}+a\frac{\partial^2 y}{\partial t^2}=q(x,c),$$

where a is the density of the beam. In this case the beam is weightless and therefore $a = 0$. On the other hand, the acceleration $\frac{\partial^2 y}{\partial t^2}$ is a finite quantity owing to the finite mass at the end of the beam. Thus

$$a\frac{\partial^2 y}{\partial t^2}=0$$

and the equation of motion assumes the form

$$EI\frac{\partial^4 y}{\partial x^4}=q(x,c) \quad (c=c(t)). \tag{3}$$

with the boundary conditions

$$1.\ y'_x(0,c)=0, \qquad 2.\ EIy_x'''(0,c)-k[y(b,c)-y(0,c)]=0,$$

$$3.\ y_x''(b,c)=0, \qquad 4.\ EIy_x'''(b,c)-k[y(b,c)-y(0,c)]-\frac{m}{2n}y_t''(b,c)=0\ ,\ ^*$$

where $k = \frac{K}{2n}$ the stiffness of the spring

If use is made of the relation: $\frac{\partial^2 y}{\partial t^2}=\frac{\partial^2 y}{\partial c^2}\left(\frac{dc}{dt}\right)^2+\frac{\partial y}{\partial c}\cdot\frac{d^2c}{dt^2}$,

the fourth condition may be rewritten in the following form:

$$4.\ EIy_x'''(b,c)-k[y(b,c)-y(0,c)]-\frac{m}{2n}y''_c(b,c)c'^2-\frac{m}{2n}y'_c(b,c)c''=0.$$

The initial conditions are: $1.\ y(b,0)=0, \quad 2.\ y_c'(b,0)c'_{c=0}=V_0.$

Denote any function of x through $\frac{-\Delta l}{EI}p(x,c)$ by $P_1(x,c)$, any function $P_1(x,c)$ by $P_2(x,c)$, and so forth.

From equation (3) there is obtained

$$\frac{\partial^3 y}{\partial x^3}=\begin{cases}P_1(x,c)+C_1(c)\ \text{at}\ [0,c],\\ P_1(c,c)+C_1(c)\ \text{at}\ [c,b],\end{cases}$$

$$\frac{\partial^2 y}{\partial x^2}=\begin{cases}P_2(x,c)+C_1(c)x+C_2(c)\ \text{at}\ [0,c],\\ P_2(c,c)+C_1(c)C+C_2(c)+[P_1(c,c)+C_1(c)](x-c)\ \text{at}\ [c,b],\end{cases}$$

$$\frac{\partial y}{\partial x}=\begin{cases}P_3(x,c)+\frac{1}{2}C_1(c)x^2+C_2(c)x+C_3(c)\ \text{at}\ [0,c],\\ P_3(c,c)+\frac{1}{2}C_1(c)c^2+C_2(c)c+C_3(c)+[P_2(c,c)+C_1(c)c+\\ \qquad +C_2(c)](x-c)+[P_1(c,c)+C_1(c)]\frac{(x-c)^2}{2}\ \text{at}\ [0,b],\end{cases}$$

*No account is taken of the weight of the concentrated mass since for a sufficient immersion of the float the mass is compensated by the hydrostatic pressure and is moreover a small magnitude by comparison with the inertia forces.

$$y=\begin{cases} P_4(x, c+\frac{1}{6}C_1(c)x^3+\frac{1}{2}C_2(c)x^2+C_3(c)x+C_4(c) \text{ at } [0, c], \\ P_4(c, c)+\frac{1}{6}C_1(c)c^3+\frac{1}{2}C_2(c)c^2+C_3(c)c+C_4(c)+[P_3(c, c)+ \\ \quad +\frac{1}{2}C_1(c)c^2+C_2(c)c+C_3(c)](x-c)+[P_2(c, c)+C_1(c)c+ \\ \quad +C_2(c)]\frac{(x-c)^2}{2}+[P_1(c, c)+C_1(c)]\frac{(x-c)^3}{6} \text{ at } [c, b]. \end{cases}$$

With the aid of these relations and after the usual transformations the boundary conditions can be written as follows:

1. $C_3(c)=-P_3(0, c).$

2. $$\left(EI-\frac{kb^3}{6}\right)C_1(c)-\frac{kb^2}{2}C_2(c)=-EIP_1(0, c)+\frac{k}{6}(b-c)^3P_1(c, c)+$$
$$+\frac{k}{2}(b-c)^2P_1(c, c)-kbP_3(0, c)+k(b-c)P_3(c, c)-kP_4(0, c)+kP_4(c, c).$$

3. $bC_1(c)+C_2(c)=-(b-c)P_1(c, c)-P_2(c, c).$

4. $$\frac{m}{2n}c'^2C_4''(c)+\frac{m}{2n}c''C_4'(c)-EI[P_1(c, c)+C_1(c)]+$$
$$+k\Bigg[P_4(c, c)-P_4(0, c)+\frac{1}{6}C_1(c)c^3+\frac{1}{2}C_2(c)c^2+C_3(c)c+\Big(P_3(c, c)+$$
$$+\frac{1}{2}C_1(c)c^2+C_2(c)c+C_3(c)\Big)(b-c)+\Big(P_2(c, c)+C_1(c)c+$$
$$+C_2(c)\Big)\frac{(b-c)^2}{2}+\Big(P_1(c, c)+C_1(c)\Big)\frac{(b-c)^3}{6}\Bigg]+\frac{m}{2n}c'^2\Bigg\{\frac{d^2}{dc^2}P_4(c, c)-$$
$$-2\frac{d}{dc}P_3(c, c)+P_2(c, c)+\frac{1}{6}C_1''(c)c^3+\frac{1}{2}C_2''(c)c^2+C_3''(c)c+$$
$$+(b-c)\left[\frac{d^2}{dc^2}P_3(c, c)-2\frac{d}{dc}P_2(c, c)+P_1(c, c)+\frac{1}{2}C_1''c^2+C_2''c+\right.$$
$$\left.+C_3''\right]+\frac{(b-c)^2}{2}\left[\frac{d^2}{dc^2}P_2(c, c)-2\frac{d}{dc}P_1(c, c)+C_1''(c)c+C_2''(c)\right]+$$
$$+\frac{(b-c)^3}{6}\left[\frac{d^2}{dc^2}P_1(c, c)+C_1''(c)\right]\Bigg\}+\frac{m}{2n}c''\Bigg\{\frac{d}{dc}P_4(c, c)-P_3(c, c)+$$
$$+\frac{1}{6}C_1'(c)c^3+\frac{1}{2}C_2'(c)c^2+C_3'(c)c+(b-c)\left[\frac{d}{dc}P_3(c, c)-P_2(c, c)+\right.$$
$$\left.+\frac{1}{2}C_1'(c)c^2+C_2'(c)c+C_3'(c)\right]+\frac{(b-c)^2}{2}\left[\frac{d}{dc}P_2(c, c)-P_1(c, c)+\right.$$
$$\left.+C_1'(c)c+C_2'(c)\right]+\frac{(b-c)^3}{6}\left[\frac{d}{dc}P_1(c, c)+C_1'(c)\right]\Bigg\}=0.$$

From the first condition $C_3(c)$ is determined and from the second and third $C_1(c)$ and $C_2(c)$. Having determined $C_1(c)$, $C_2(c)$, and $C_3(c)$, the coefficient $C_4(c)$ is determined from the fourth boundary condition and since this condition is a differential equation of the second order with respect to $C_4(c)$ the latter is obtained with two arbitrary constants the value of which is found from two initial conditions. But $C_4(c)$ need not be determined because the normal stress σ is expressed through the bending moment in the expression for which $C_4(c)$ does not enter. Thus by making use of the expression for $\frac{\partial^2 y}{\partial x^2}$ the following formula is obtained for the bending moment M:

$$M = \begin{cases} EI\,[P_2(x,c) + C_1(c)\,x + C_2(c)] & \text{at } [0, c], \\ EI\,[P_2(c,c) + C_1(c)\,c + C_2(c) + (P_1(c,c) + C_1(c))(x - c)] & \text{at } [c, b]. \end{cases} \tag{4}$$

From the second and third boundary conditions an expression is obtained for the values of $C_1(c)$ and $C_2(c)$:

$$C_1(c) = \Big[-EIP_1(0,c) + \frac{1}{6}k(b-c)(c^2 - 2bc - 2b^2)\,P_1(c,c) + \\ + \frac{1}{2}kc\,(c - 2b)\,P_2(c,c) - kbP_3(0,c) + \\ + k(b-c)\,P_3(c,c) - kP_4(0,c) + kP_4(c,c)\Big]\frac{1}{EI + \frac{kb^3}{3}}. \tag{5}$$

$$C_2(c) = \Big[EIbP_1(0,c) - (b-c)\Big(EI - \frac{kbc^2}{3} + \frac{kbc^2}{6}\Big)P_1(c,c) - \\ - \Big(EI + \frac{kb^3}{3} - kbc^2 + \frac{kbc^2}{2}\Big)P_2(c,c) + kb^2P_3(0,c) - \\ - kb(b-c)\,P_3(c,c) + kbP_4(0,c) - kbP_4(c,c)\Big]\frac{1}{EI + \frac{kb^3}{3}}. \tag{6}$$

Integrating successively with respect to x the function $-\frac{\Delta l}{EI}\,p(x,c)$ results in the following expressions for $P_1(x,c)$, $P_2(x,c)$, $P_3(x,c)$, and $P_4(x,c)$:

$$P_1(x,c) \doteq \frac{-V_0^2\Delta l}{EI(1+\mu)^2}\,\frac{\rho}{u}\Big\{c\Big[1 + u^2 + cu\frac{du}{dc} - \frac{\mu}{1+\mu}(1+u^2)\Big]\arcsin\frac{x}{c\sqrt{2+u^2}} - \\ - \frac{\mu}{1+\mu}\,x\sqrt{1 + u^2 - \frac{x^2}{c^2}} + \frac{ux}{2} - \frac{uc}{4}\sqrt{1+u^2}\,\ln\frac{c\sqrt{1+u^2}+x}{c\sqrt{1+u^2}-x}\Big\},$$

$$P_2(x,c) = \frac{-V_0^2\,\Delta l}{EI(1+\mu)^2}\,\frac{\rho}{u}\Big\{c\Big(1 + u^2 + cu\frac{du}{dc}\Big)\Big(x\arcsin\frac{x}{c\sqrt{1+u^2}} +$$

$$+c\sqrt{1+u^2-\frac{x^2}{c^2}}\Bigg)-\frac{\mu}{1+\mu}\left[\frac{2c^2(1+u^2)+x^2}{3}\sqrt{1+u^2-\frac{x^2}{c^2}}+\right.$$

$$\left.+c(1+u^2)x\arcsin\frac{x}{c\sqrt{1+u^2}}\right]-\frac{u}{4}\left[-x^2-2c^2(1+u^2)+\right.$$

$$\left.\left.+c^2(1+u^2)\ln\left(1-\frac{x^2}{c^2(1+u^2)}\right)+c\sqrt{1+u^2}\ln\frac{c\sqrt{1+u^2}+x}{c\sqrt{1+u^2}-x}\right]\right\},$$

$$P_3(x,c)=\frac{-V_0^2\,\Delta l}{EI(1+\mu)^2}\,\frac{\rho}{u}\left\{\frac{c}{4}\left(1+u^2+cu\frac{du}{dc}\right)\left[\left(2x^2+\right.\right.\right.$$

$$\left.\left.+c^2(1+u^2)\right)\arcsin\frac{x}{c\sqrt{1+u^2}}+3xc\sqrt{1+u^2-\frac{x^2}{c^2}}\right]-$$

$$-\frac{\mu}{2(1+\mu)}\left[c(1+u^2)\left(x^2+\frac{c^2(1+u^2)}{4}\right)\arcsin\frac{x}{c\sqrt{1+u^2}}+\right.$$

$$\left.+\frac{x}{6}\left(\frac{13}{2}c^2(1+u^2)+x^2\right)\sqrt{1+u^2-\frac{x^2}{c^2}}\right]-\frac{u}{4}\left[-\frac{x^3}{3}-3c^2(1+u^2)x+\right.$$

$$+\frac{c}{2}\sqrt{1+u^2}\,(c\sqrt{1+u^2}+x)^2\ln\left(1+\frac{x}{c\sqrt{1+u^2}}\right)-$$

$$\left.\left.-\frac{c}{2}\sqrt{1+u^2}\,(c\sqrt{1+u^2}-x)^2\ln\left(1-\frac{x}{c\sqrt{1+u^2}}\right)\right]\right\};$$

$$P_4(x,c)=\frac{-V_0^2\,\Delta l}{EI(1+\mu)^2}\,\frac{\rho}{u}\left\{\frac{1}{2}c\left(1+u^2+cu\frac{du}{dc}\right)\left[\left(\frac{c^2(1+u^2)}{2}+\frac{x^2}{3}\right)\times\right.\right.$$

$$\times\left(x\arcsin\frac{x}{c\sqrt{1+u^2}}+c\sqrt{1+u^2-\frac{x^2}{c^2}}\right)-$$

$$\left.-\frac{5}{18}\left(c^2(1+u^2)-x^2\right)c\sqrt{1+u^2-\frac{x^2}{c^2}}\right]-\frac{\mu}{6(1+\mu)}\times$$

$$\times\left[c(1+u^2)x\left(x^2+\frac{3}{4}c^2(1+u^2)\right)\arcsin\frac{x}{c\sqrt{1+u^2}}+\right.$$

$$\left.+\left(\frac{4}{15}c^4(1+u^2)^2+\frac{83}{60}c^2(1+u^2)x^2+\frac{1}{10}x^4\right)c\sqrt{1+u^2-\frac{x^2}{c^2}}\right]-$$

$$-\frac{u}{24}\left[-x^4-11c^2(1+u^2)x^2+c\sqrt{1+u^2}(c\sqrt{1+u^2}+x)^3\ln\left(1+\frac{x}{c\sqrt{1+u^2}}\right)+\right.$$

$$\left.\left.+c\sqrt{1+u^2}\,(c\sqrt{1+u^2}-x)^3\ln\left(1-\frac{x}{c\sqrt{1+u^2}}\right)\right]\right\}$$

Substituting these expressions in formulas (6), (5), and (4) gives the bending moment distribution along the beam. However, the value of the moment at the critical section is of interest. For a float bottom with weak keel (such as is usually the case in present day designs) the critical section will be at the point $x = 0$.

Thus

$$M_0(c) = \frac{-V_0^2 \Delta l \rho c^2}{u(1+\mu)^2}\left[\sqrt{1+u^2}\left(\frac{1+u^2}{3} + \frac{2}{3}\frac{1+u^2}{1+\mu} + cu\frac{du}{dc}\right) + \right.$$

$$\left. + \frac{u(1+u^2)}{2} + A(c)\right], \tag{7}$$

is found, where

$$A(c) = \frac{EI}{c^2}\frac{u(1+\mu)^2}{V_0^2 \Delta l \rho c^2} C_2(c) =$$

$$= b\left[\left\{-\left(1+u^2+cu\frac{du}{dc}\right)\left[EI + \frac{kbc^2}{4}(1+u^2)\right] + \frac{\mu}{1+\mu}(1+u^2)\left[EI + \right.\right.\right.$$

$$\left.\left. + \frac{kbc^2}{8}(1+u^2)\right]\right\}\arcsin\frac{1}{\sqrt{1+u^2}} + \frac{kbc^3(1+u^2)\sqrt{1+u^2}}{9}\left[1+u^2+cu\frac{du}{dc} - \right.$$

$$\left. - \frac{2}{5}\frac{\mu}{1+\mu}(1+u^2)\right] - \frac{u(b-c)(1-\mu)}{2(1+\mu)}\left[EI - \frac{kb^2c}{3} + \frac{kbc^2}{6}\right] -$$

$$- cu\left[EI + \frac{kb^3}{3} - kb^2c + \frac{kbc^2}{2}\right]\left[\frac{9+10u^2}{12} + \frac{3+2u^2}{3(1+\mu)} + cu\frac{du}{dc}\right] -$$

$$- \frac{1}{4}c^2ukb(b-c)\left[\frac{28+23u^2}{6} + \frac{10+13u^2}{6(1+\mu)} + 3cu\frac{du}{dc}\right] -$$

$$- \frac{1}{36}c^2ukb\left[(15+4u^2)cu\frac{du}{dc} + \frac{225+240u^2+24u^4}{10} + \right.$$

$$\left. + \frac{105+115u^2+16u^4}{10(1+\mu)}\right] + \frac{u\sqrt{1+u^2}}{4}\left\{(b-c)\left[EI - \frac{kb^3}{6} + \frac{kb(b-c)^2}{6} + \right.\right.$$

$$\left. + \frac{1}{2}kbc^2(2+u^2)\right] + c\left[EI - \frac{kb^3}{6} + \frac{kb(b-c)^2}{2} + \right.$$

$$\left.\left. + \frac{kbc^2}{6}(4+3u^2)\right]\right\}\ln\frac{\sqrt{1+u^2}+1}{\sqrt{1+u^2}-1} +$$

$$+ \frac{uc(1+u^2)}{4}\left[EI - \frac{kb^3}{6} + \frac{kb(b-c)^2}{2} + kb(b-c)c + \right.$$

$$\left.\left. + \frac{1}{6}kbc^2(4+u^2)\right]\ln\left(1 - \frac{1}{1+u^2}\right)\right]\frac{1}{EIc + \frac{kb^3c}{3}} \tag{8}$$

The value of u is, however, sufficiently small (for a wedge with straight sides $u = \frac{2}{\pi}\tan\beta$), so that in most of the terms of equations (7) and (8) u^2 and $cu\frac{du}{dc}$ may be neglected.

Finally, there is obtained

$$M_o(c) = -\frac{V_o^2 \Delta l \rho c^2}{u(1 + \mu)^2}\left[\frac{3 + \mu}{3(1 + \mu)} + \frac{u}{2} + A(c)\right] \tag{9}$$

$$A(c) = \Big[-\frac{b}{720(1 + \mu)}[360\pi EI + 10kc^2(9\pi b - 8c) +$$

$$\mu kc^2(45\pi b - 48c)] - \frac{u}{24(1 + \mu)}\Big\{6EI[2b(1 - \mu) + 5c(1 + \mu)] +$$

$$kbc[10b^2(1 + \mu) + 2bc(1 + 2\mu) + c^2(3 - 2\mu)]\Big\} + \frac{ub}{8}(2EI +$$

$$kbc^2)\, ln \frac{2}{\sqrt{1 + u} - 1} + \frac{uc}{24}[6EI + kb(b^2 +$$

$$c^2)]\, ln\left(1 - \frac{1}{1 + u^2}\right)\Big]\frac{1}{EIc + \frac{kb^3c}{3}} \tag{10}$$

The following should be remarked. In the square brackets of formula (10), the second, third, and fourth terms, notwithstanding the presence of the factor u, may attain approximately a value of 20 percent of that of the first term. It may be shown, however, that for the usual bottom shapes and in the case of a weak keel, where the stiffness of the latter may be of the same order as the stiffness of the strongest floor beam, the algebraic sum of the preceding terms is small by comparison with the first term and constitutes not more than 5 percent for $c \leqslant \frac{b}{2}$ and not more than about 10 percent for $c > \frac{b}{2}$. On this basis equation (10) may be replaced by the equation

$$A(c) = -\frac{b}{720(1 + \mu)}[360\pi EI + 10kc^2(9\pi b - 8c) +$$

$$\mu kc^2(45\pi b - 48c)]\frac{1}{EIc + \frac{kb^3c}{3}} \tag{10'}$$

which will be sufficiently accurate for $c \leqslant \frac{b}{2}$ and will give an error of the order of 10 percent for $c > \frac{b}{2}$.

By formulas (9) and (10) or (9) and (10') the stresses in the floor as a function of c are determined. To obtain the maximum stress, a curve $M_0(c)$ is drawn, a few values of the bending moment being determined by (9), and the maximum value of $M_0(c)$ obtained. This value should be obtained, in general, for a certain intermediate value of c. Thus, since the impact force increases with increase in c, the bending moment $M_0(c)$ will likewise at first increase. On the other hand, since with increasing c the force of the hydrodynamic impact is concentrated usually at the edge of the wetted surface, it should be expected that after a certain instant of time the value of $M_0(c)$ should begin to decrease, particularly since with increasing c the reaction of the keel increases (fig. 6).

For V angles exceeding 15° the right side of equation (1), and therefore also of equation (9), should be multiplied by the correction factor ν:

$$\nu = 1 - \frac{\beta'}{\pi} - 0.15\,\frac{u}{\pi} - \frac{u}{\pi}\,ln\,\frac{1}{u}$$

where β' is the angle, in radians, of inclination of the bottom at the edge of the impact surface.

Moreover equation (1) was obtained for the conditions of the two-dimensional problem; that is, the wedge was assumed as infinitely long. Practically, equation (1) may be used when the length of the wedge is sufficiently large by comparison with the width, at least one and one-half times as large. As has already been remarked at the beginning of this paper, for landings on the bow and on the stern the length of the impact surface is sufficiently great and may be obtained by considering the landing on a calm surface.

For landing on the step the most unfavorable condition should be considered - namely, when the float lands on a wave. As experiments have shown, the length of the impact surface then exceeds the width by 1.5 and more times. Since, for a ratio of length of wedge to width near 1.5 the error in computing the impact force on the assumption of two-dimensional flow is appreciable, a correction for finiteness is desirable.

This correction may be made with the aid of the experimental formulas obtained by Pabst for the associated mass of flat plates and by Povitsky for the associated mass of wedges. For $\frac{l}{b} \geqslant 1.5$, and usual V angles, the formula of Pabst differs little from that of Povitsky. Both of them will be given.

Let b be the half width of the plate and l its length. The associated mass M for the plate for the assumption of two-dimensional flow is

$$M = \frac{\pi}{2}\rho l c^2 \tag{11}$$

the associated mass for the finite plate $\left(\frac{l}{2c} \geqslant 1\right)$ is:

$$M = \frac{\pi}{2}\rho(lc^2 - c^3) \tag{12}$$

For an element of an infinitely long wedge the associated mass is

$$M = \frac{\delta}{\cos^2\beta}\rho l c^2 \tag{13}$$

where

l length of element of wedge

c half width of wetted surface of wedge

β V angle

$$\delta = \frac{1}{2} \sin 2\beta \left[- \frac{\Gamma\left(\frac{3}{2} - \frac{\beta}{\pi}\right) \Gamma\left(\frac{\beta}{\pi}\right)}{\Gamma\left(\frac{1}{2} + \frac{\beta}{\pi}\right) \Gamma\left(1 - \frac{\beta}{\pi}\right)} - 1 \right]$$

(Γ denotes the Gamma function)

The value of δ also may be obtained from the curve of figure 8.

For the wedge of finite length $l\left(\frac{l}{2d} \geqslant 1.425\right.$, where d is the length of the wetted part of the side of the wedge$\left.\right)$

$$M = \frac{\delta}{\cos^2\beta}\rho l c^2 \left(1 - \frac{c}{l}\right) \tag{14}$$

The impact force for the wedge is expressed by the formula

$$P = \frac{V_o^2}{\left(1 + \frac{M}{m}\right)^3 u} \frac{dM}{dc}$$

If the impact force for the case of the two-dimensional problem is denoted by P_{inf} and the true impact force for the finite wedge by P_t, then

$$\frac{P_t}{P_{inf}} = \left(\frac{m + M_{inf}}{m + M_t}\right)^3 \frac{\frac{dM_t}{dc}}{\frac{dM_{inf}}{dc}} = \varphi(c)$$

Without taking into account the pressure distribution along the width of the wedge (the pressure distribution is not identical for the infinite and finite wedges) a correction is made for finiteness by multiplying the pressure determined by formula (1) by $\varphi(c)$. In formula (9) it is then likewise necessary to multiply the right side by $\varphi(c)$. By use of formulas (11) and (12), there is obtained

$$\varphi(c) = \frac{1}{\left(1 - \frac{1}{\frac{2m}{\pi\rho c^3} + \frac{l}{c}}\right)^3} \left(1 - \frac{3}{2}\frac{c}{l}\right) \tag{15}$$

From (13) and (14) is obtained

$$\varphi = \frac{1}{\left(1 - \frac{1}{\frac{\cos^2}{\delta\rho}\frac{\beta m}{c^3} + \frac{l}{c}}\right)^3} \left(1 - \frac{3}{2}\frac{c}{l}\right) \tag{16}$$

Thus, for computing the bending moment M and the correction coefficient $\varphi(c)$ for landing on the step, it remains merely to determine the length of the impact surface l. It should be noted that, during the immersion of the float in the water, the length l is, in general, variable. It is nevertheless possible to assume a certain constant value for l since, in the most unfavorable case - namely, landing on a wave - the float almost instantaneously comes in contact with the wave over a considerable length of surface which afterward increases only slightly. For the same reason it is of no significance to make any correction for finiteness at the first instants of immersion. The correction should be made when a sufficient width of the surface has already been immersed.

The character of the sea surface permissible for the landing of seaplanes varies with the tonnage. The following mean values for the limiting height of the wave h in relation to aircraft of various tonnage ranges may be given (reference 6):

First group	G = 700 - 1500 kilograms,	h = 0.15 - 0.3 meter
Second group	G = 3500 - 5000 kilograms,	h = 0.8 - 1.0 meter
Third group	G = 8000 - 15000 kilograms,	h = 1.5 - 1.8 meters
Fourth group	G = 20,000 and above,	h = 2.0 meters

With improvement in the seaway the preceding values may be raised somewhat. The length of the wave λ is determined as a function of h by the curve of Zimmerman (reference 7). The wave is sufficiently well represented by a trochoid. The equation of the trochoid in parametric form is expressed by

$$x = \frac{\lambda}{2\pi}\varphi + \frac{h}{2}\sin\varphi \quad y = \frac{h}{2} - \frac{h}{2}\cos\varphi$$

where λ is the length of the wave and h its height. The trochoid also may be obtained by a simple geometric construction (reference 8). The latter is indicated by the thin lines of figure 9. For the trim angle the mean angle of inclination of the tangent to the wave increased by 3° to 4° may be taken. For aircraft of not too great tonnage (2500 to 5000 kg) the condition of the sea may be taken as a "calm surface" (the wave intensity is 2 points) which corresponds to an average force of the wing equal approximately to 3 meters per second (2.8 m/sec at a height of 2 m from ground level and 3.6 m/sec at a height of 12 m from ground level). Under these conditions the height of the wave is 1 meter and its length 11 meters. The angle of inclination of the tangent to such a wave varies from zero to 15° so that the trim angle may be taken as 10°.

To determine the length of the wetted surface, draw the contour of the float with the chosen trim angle on the profile of the wave, immersing the step to the chine (fig. 10). It is necessary, of course, to see that the tail of the float does not dig into the wave (this refers especially to floats with long tails). The maximum length of the wetted surface is taken as the value of l.

As tests have shown, in landing on the step the pressures at the left end of the wetted surface are found to be negligible. This may be explained by the fact that because of the longitudinal curvature of the bottom the points sufficiently far removed from the step come in contact with the water at later instants of time. Moreover, the distance of these points from the center of gravity of the aircraft apparently has some effect. For this reason, for the purpose of greater accuracy, it is necessary to take for l not the entire length of the impact surface but a smaller value - namely, the length of the part at which the bottom is almost cylindrical.

If the width of the float is not too great, it may be assumed that the maximum impact force will occur at immersion of the chine. Since, however, the ratio of the width of the wetted surface to the length after the instant when the float is already sufficiently immersed in the water varies little, the value of $\varphi(c)$ may be assumed constant with the width of the float taken as the width of the impact surface.

ILLUSTRATIVE EXAMPLE

The computation is conducted for a symmetrical landing on the step. The fundamental data required for the computation are the following:

G weight in flight (2500 kg)

V_l landing velocity (30.6 m/sec)

b half width of float at step (75 cm)

A sketch of part of keel with bulkheads lying beyond the impact surface is shown in figure 11 (the length of the wetted surface must be initially determined).

I_k mean moment of inertia of part of keel beam under consideration (560 cm^4)

I_Φ mean moment of inertia of floors lying directly beyond impact surface (90 cm^4)

I mean moment of inertia of floors associated with impact surface (313 cm^4)

W mean resistance moment of floors associated with impact surface (20.85 cm^3)

E elasticity modulus (7.2×10^5 kg/cm^2)

Computation

Computation of κ.– $\varkappa = \frac{48EI_\phi}{l^3} = \frac{48 \cdot 7{,}2 \cdot 10^5 \cdot 90}{140^3} = 15{,}88 \cdot 10^2 \ \frac{\text{kg}}{\text{cm}}$

Computation of k.– For the deflection and angular displacement at section X of a beam on elastic supports under the action of a force P applied at the center of beam, the following formulas are obtained: (Reference 4 makes use of the same notation.)

$$y_x = y_0 A_x + \varphi_0 B_x - M_0 C_x - Q_0 D_x + N_x,$$

$$\varphi_x = \varphi_0 A_x' - M_0 B_x' - Q_0 C_x' - y_0 D_x' + N_x,$$

where y_o, φ_o, M_o, and Q_o are the deflection, angular displacement, bending moment, and shearing force at the origin of coordinates.

The values A, B, C, D, N, A', B', C', D', N' at the nth support (from the origin) are determined by the recurrence formulas:

$$A_n = 1 - \sum_{i=1}^{n-1} \varkappa_i A_i \hat{I}_{n-i},$$

$$B_n = x_n - \sum_{i=1}^{n-1} \varkappa_i B_i \hat{I}_{n-i},$$

$$C_n = \hat{S}_n - \sum_{i=1}^{n-1} \varkappa_i C_i \hat{I}_{n-i},$$

$$D_n = \hat{I}_n - \sum_{i=1}^{n-1} \varkappa_i D_i \hat{I}_{n-i},$$

$$N_n = P\hat{I}_{n-p} - \sum_{i=1}^{n-1} \varkappa_i N_i \hat{I}_{n-1}$$

$$\left(\hat{S}_n = \frac{X_n^2}{2EI}; \qquad \hat{I}_{n-i} = \frac{(x_n - x_i)^3}{6EI}\right)$$

and

$$A_n' = 1 - \sum_{i=1}^{n-i} \varkappa_i B_i S_{n-i},$$

$$B_n' = F_n - \sum_{i=1}^{n-1} \varkappa_i C_i S_{n-i},$$

$$C_n' = S_n - \sum_{i=1}^{n-1} \varkappa_i D_i S_{n-i},$$

$$D_n' = \sum_{i=1}^{n-1} \varkappa_i A_i S_{n-1},$$

$$N_n' = P\ S_{n-p} - \sum_{i=1}^{n-i} \varkappa_i N_i S_{n-i}$$

$$\left(S_{n-i} = \frac{(x_n - x_i)^2}{2EI}, \qquad F_n = \frac{x_n}{EI}\right).$$

Place the origin of coordinates at the left built-in end. (See sketch of part of keel under consideration.)
Then $y_0 = \varphi_0 = 0$

and in the formulas for y_x and φ_x there only enter the values C, D, N and B', C', N'.

The values of M_0 and Q_0 are obtained by equating to zero the expression for y and φ at the right end. Since y and φ for the right end are no other than y_5, φ_5, the magnitudes C_1, C_2, C_3, C_4, C_5, D_1, D_2, D_3, D_4, D_5, N_1, N_2, N_3, N_4, N_5 are determined in succession.

Then
$$C_1 = \hat{S}_1 = \frac{40^2}{2\cdot 40{,}3\cdot 10^7} = 1{,}98\cdot 10^{-6}$$

$$C_2 = \hat{S}_2 - \varkappa C_1\hat{I}_{2-1} = \frac{80}{2\cdot 40{,}3\cdot 10^7} - 15{,}88\cdot 10^2\cdot 1{,}98\cdot 10^{-6}\cdot\frac{40^3}{6\cdot 40{,}3\cdot 10^7} = 7{,}85\cdot 10^{-6}$$

$$C_3 = \hat{S}_3 - \varkappa C_1\hat{I}_{3-1} - \varkappa C_2\hat{I}_{3-2} = \frac{350^2}{2\cdot 40{,}3\cdot 10^7} - 15{,}88\cdot 10^2 1{,}98\cdot 10^{-6}\cdot\frac{310^3}{6\cdot 40{,}3\cdot 10^7} -$$
$$- 15{,}88\cdot 10^2\cdot 7{,}85\cdot 10^{-6}\frac{270^3}{6\cdot 40{,}3\cdot 10^7} = 12{,}05\cdot 10^{-6}$$

$$C_4 = \hat{S}_4 - \varkappa C_1\hat{I}_{4-1} - \varkappa C_2\hat{I}_{4-2} - \varkappa C_3\hat{I}_{4-3} = \frac{390^2}{2\cdot 40{,}3\cdot 10^7} -$$
$$- 15{,}88\cdot 10^2\cdot 1{,}98\cdot 10^{-6}\cdot\frac{350^3}{6\cdot 40{,}3\cdot 10^7} - 15{,}88\cdot 10^2\cdot 7{,}85\cdot 10^{-6}\cdot\frac{310^3}{6\cdot 40{,}3\cdot 10^7} -$$
$$- 15{,}88\cdot 10^2\cdot 12{,}05\cdot 10^{-6}\frac{40^3}{6\cdot 40{,}3\cdot 10^7} = -20{,}093\cdot 10^{-6}$$

$$C_5 = \hat{S}_5 - \varkappa C_1\hat{I}_{5-1} - \varkappa C_2\hat{I}_{5-2} - \varkappa C_3\hat{I}_{5-3} - \varkappa C_4\hat{I}_{5-4} =$$
$$= \frac{430^2}{2\cdot 40{,}3\cdot 10^7} - 15{,}88\cdot 10^2\cdot 1{,}98\cdot 10^{-6}\frac{390^3}{6\cdot 40{,}3\cdot 10^7} -$$
$$- 15{,}88\cdot 10^2\cdot 7{,}85\cdot 10^{-6}\frac{350^3}{6\cdot 40{,}3\cdot 10^7} - 15{,}88\cdot 10^2\cdot 12{,}05\cdot 10^{-6}\frac{80^3}{6\cdot 40{,}3\cdot 10^7} +$$
$$+ 15{,}88\cdot 10^2\cdot 20{,}093\cdot 10^{-6}\frac{40^3}{6\cdot 40{,}3\cdot 10^7} = -73{,}3\cdot 10^{-6}.$$

In the same manner, by the preceding formulas the remaining magnitudes are computed. The results are given in the following table:

	1	2	3	4	5
C	$1{,}98\cdot 10^{-6}$	$7{,}85\cdot 10^{-6}$	$12{,}05\cdot 10^{-6}$	$-20{,}093\cdot 10^{-6}$	$-73{,}3\cdot 10^{-6}$
D	$26{,}45\cdot 10^{-6}$	$210{,}8\cdot 10^{-6}$	$14510\cdot 10^{-6}$	$19570\cdot 10^{-6}$	$20350\cdot 10^{-6}$
N	0	0	$1016\cdot 10^{-6}\,P$	$2168\cdot 10^{-6}\,P$	$3682\cdot 10^{-6}\,P$
B'	—	—	—	—	$-1{,}504\cdot 10^{-6}$
C'	—	—	—	—	$-73{,}6\cdot 10^{-6}$
N'	—	—	—	—	$37{,}7\cdot 10^{-6}\cdot P$

Now M_0, Q_0 are determined from the equations:

$$y_5 = 73{,}3\cdot10^{-6}M_0 - 20350\cdot10^{-6}Q_0 + 3682\cdot10^{-6}P,$$
$$\varphi_5 = 1{,}504\cdot10^{-6}M_0 + 73{,}6\cdot10^{-6}Q_0 + 37{,}7\cdot10^{-6}P.$$

There is obtained

$$M_0 = -\frac{\begin{vmatrix} 3682 & -20350 \\ 37{,}7 & 73{,}6 \end{vmatrix}}{\begin{vmatrix} 73{,}3 & 20350 \\ 1{,}504 & 73{,}6 \end{vmatrix}} P = -29{,}2P,$$

$$Q_0 = -\frac{\begin{vmatrix} 73{,}3 & 3682 \\ 1{,}504 & 37{,}7 \end{vmatrix}}{\begin{vmatrix} 73{,}3 & 20350 \\ 1{,}504 & 73{,}6 \end{vmatrix}} P = 0{,}0769P.$$

Then the maximum deflection is found. The coordinate of the center of the beam is

$$x_p = 215 \text{ cm}$$

Since

$$C_{x_p} = \hat{S}_{x_p} - \varkappa C_1 \hat{I}_{x_p - x_1} - \varkappa C_2 \hat{I}_{x_p - x_2} =$$
$$= \frac{215^2}{2\cdot40{,}3\cdot10^7} - 15{,}88\cdot10^2\cdot1{,}98\cdot10^{-6}\frac{175^3}{6\cdot40{,}3\cdot10^7} -$$
$$-15{,}88\cdot10^2\cdot7{,}85\cdot10^{-6}\frac{135^3}{6\cdot40{,}3\cdot10^7} = 37{,}7\cdot10^{-6},$$

$$D_{x_p} = \hat{I}_{x_p} - \varkappa D_1 \hat{I}_{x_p - x_1} - \varkappa D_2 \hat{I}_{x_p - x_2} =$$
$$= \frac{215^3}{6\cdot40{,}3\cdot10^7} - 15{,}88\cdot10^2\cdot 26{,}45\cdot10^{-6}\frac{175^3}{6\cdot40{,}3\cdot10^7} -$$
$$-15{,}88\cdot10^2\cdot210{,}8\cdot10^{-6}\cdot\frac{135^3}{6\cdot40{,}3\cdot10^7} = 3682\cdot10^{-6},$$

$$N_{x_p} = 0$$

and therefore $\quad y_{x_p} = 29{,}2\cdot P\cdot37{,}7\cdot10^{-6} - 0{,}0769\cdot P\cdot3682\cdot10^{-6} = 845\cdot10^{-6}P.$

K is determined $\quad K = \frac{1}{845}\cdot10^6\,\frac{\text{kg}}{\text{cm}} = 11{,}83\cdot10^2\,\frac{\text{kg}}{\text{cm}}.$

Since five bulkheads are associated with the wetted surface

$$k = \frac{K}{10} = 1{,}183\cdot10^2\,\frac{\text{kg}}{\text{cm}}.$$

Determination of the function u(c).- To determine the function u(c), it is necessary first to write down an analytical expression for the bottom profile. In this case the bottom profile consists of a section of a straight line and part of a circle, but it is more convenient to express it by a curve of the form $\eta = \beta_0\xi - \beta_{n-1}\xi^n$. For $n = 4$, this curve very accurately represents the profile. The coefficient β_0 and β_3 are determined by two points of the profile.

Denoting by ξ_1 and ξ_2, respectively, the coordinates of the end of the straight line and the end of the curvilinear parts of the profile gives from the sketch

$$\xi_1 = 41 \text{ cm}, \quad \eta_1 = 14{,}4 \text{ cm},$$
$$\xi_2 = 75 \text{ cm}, \quad \eta_2 = 20 \text{ cm}.$$

whence
$$\beta_0 = \frac{\eta_1\xi_2^4 - \eta_2\xi_1^4}{\xi_1\xi_2^4 - \xi_2\xi_1^4} = \frac{14{,}4\cdot 75^4 - 20\cdot 41^4}{41\cdot 75^4 - 75\cdot 41^4} = 0{,}37;$$

$$\beta_3 = -\frac{\eta_2\xi_1 - \eta_1\xi_2}{\xi_1\xi_2^4 - \xi_2\xi_1^4} = -\frac{20\cdot 41 - 14{,}4\cdot 75}{41\cdot 75^4 - 75\cdot 41^4} = 0{,}2375\cdot 10^{-6}\left[\frac{1}{\text{cm}^3}\right]$$

and since
$$k_0 = 0{,}636, \quad k_3 = 1{,}500,$$

therefore
$$u(c) = k_0\beta_0 - k_3\beta_3 c^3 = 0{,}636\cdot 0{,}37 - 1{,}5\cdot 0{,}2375\cdot 10^{-6} c^3 =$$
$$= 0{,}235 - 0{,}356\cdot 10^{-6} c^3.$$

Computation of l, Δl, $\mu(c)$, V_0.- To determine l, the wave is drawn to a certain scale in the form of a trochoid of height of 1 meter and length 11 meters. To the same scale the contour of the float is drawn and superposed on the wave with a trim angle equal to 10° (fig. 10). Then by the same procedure as indicated previously, there is obtained from the sketch

$$l = 200 \text{ cm}$$

$$\Delta l = \frac{l}{n} = \frac{200}{5} = 40 \text{ cm}$$

Assume for ρ the value $\rho = \dfrac{0{,}001028}{g}\ \dfrac{\text{kg}}{\text{cm}^3}$,

There is obtained $\mu = \rho\dfrac{\pi l}{2m_r}c^2 = 0{,}001028\cdot\dfrac{3{,}14\cdot 200}{2\cdot 2200}c^2 = 1{,}47\cdot 10^{-4}\cdot c^2$

$$V_0 = V_{noc}\sin 10^\circ = 30{,}6\cdot 0{,}17365\frac{\text{m}}{\text{sec}} \cong 5{,}3\frac{\text{m}}{\text{sec}}$$

where
$$m_r = m\frac{i^2}{i^2 + a^2} = \frac{2\,200}{g}\ \text{kg}$$

Collection of data required for the computation.-

$$b = 75 \text{ cm}, \ \Delta l = 40 \text{ cm}$$

$$\rho = \frac{0{,}001028}{g}\ \frac{\text{kg}}{\text{cm}^3}, \ \mu = 1{,}47\cdot 10^{-4}\cdot c^2,$$

$$u = 0{,}235 = 0{,}356\cdot 10^{-6} c^3, \quad EI = 22{,}57\cdot 10^{10}\frac{\text{kg}}{\text{cm}^2},$$

$$k = 1{,}183\cdot 10^2\frac{\text{kg}}{\text{cm}}, \quad V_0 = 5{,}3\frac{\text{m}}{\text{sec}}$$

Determination of $A(c)$ and $M(c)$.- By formulas (9) and (10) are computed the values of $M_0(c)$ for a number of values of c. A curve is then drawn from which the maximum value $M_0(c)$ is determined.

For $c = \frac{b}{2} = 37.5$ centimeters, there is obtained

$$\mu = 1.47 \times 10^{-4} \times 37.5^2 = 0.2065$$

$$u = 0.235 - 0.356 \times 10^{-6} \times 37.5^3 = 0.2162$$

$$A(37.5) = -2.56$$

$$M_0(37.5) = 8.2 \times 10^4 \text{ kilogram-centimeter}$$

By computing also $M_0(c)$ for the points $c = 20, 30, 50, 75$ centimeters, a curve $M = M_0(c)$ is obtained (fig. 12). From the figure it is seen that $M_0(c)$ reaches a maximum approximately for $c = \frac{b}{2}$ and $M_0(c) \cong 8.2 \times 10^4$ kilogram-centimeter.

Determination of the correction coefficient.-

$$\varphi = \frac{1}{\left(1 - \dfrac{1}{\dfrac{2 \times 2200}{3.14 \times 1.028 \times 10^{-3} \times 75^3} + \dfrac{200}{75}}\right)^3} \left(1 - \frac{3}{2} \times \frac{75}{200}\right) = 0.766$$

Determination of the stresses σ.- Since $\varphi \times$ maximum $M_0(c) = 6.28 \times 10^4$ kilogram-centimeter and the resistance moment of the floors $W = 20.85$ cubic centimeters, there is obtained

$$\sigma = \frac{\varphi \times \max M_0(c)}{W} = 30.1 \frac{\text{kg}}{\text{mm}^2}$$

It may be noted that the present-day factory computation (the so-called "static computation") of the floor as an element of the transverse system gives $\sigma = 33$ kilograms per square millimeter.

Translation by S. Reiss,
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for Aeronautics.

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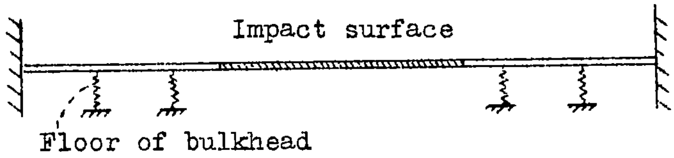
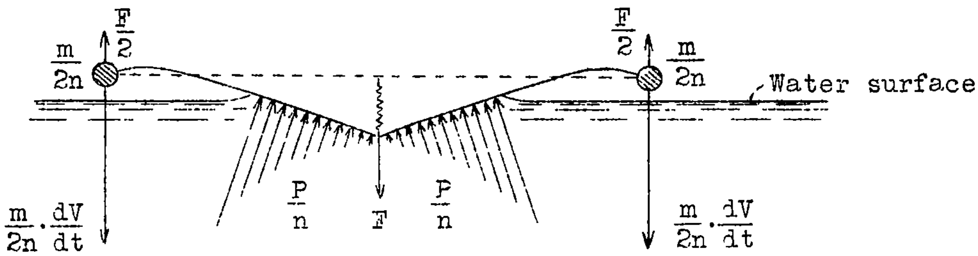


Figure 1.



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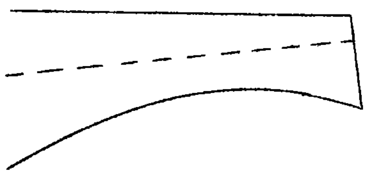

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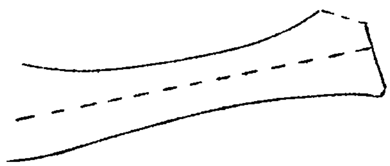

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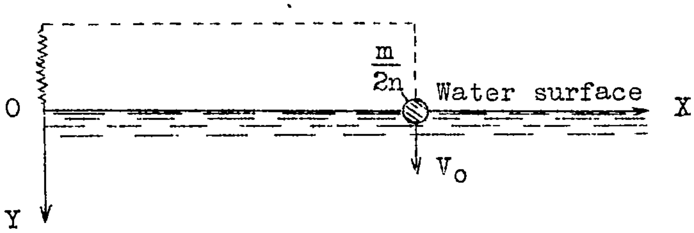

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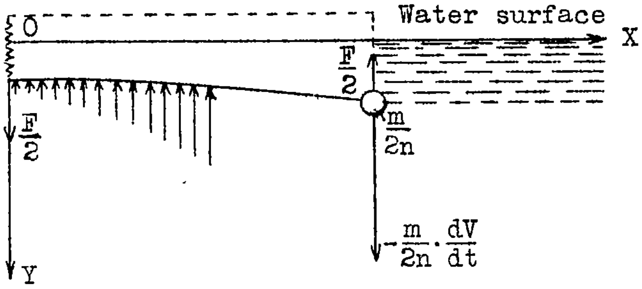

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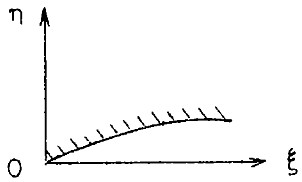

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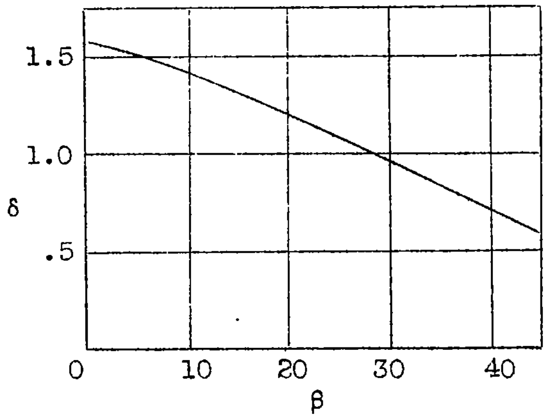

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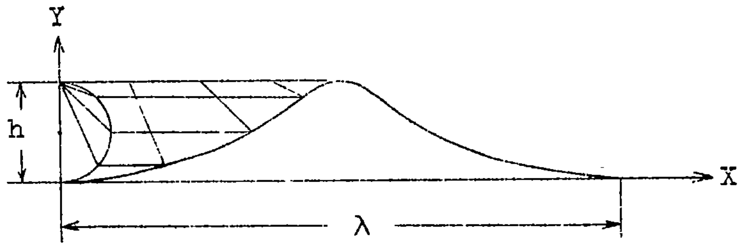

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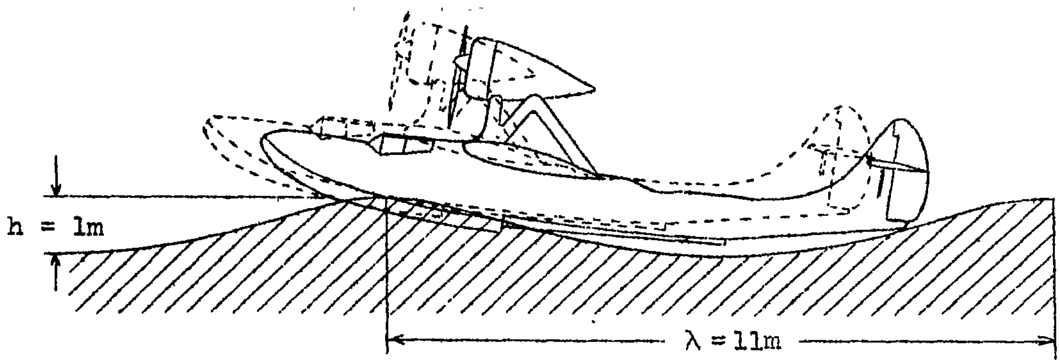

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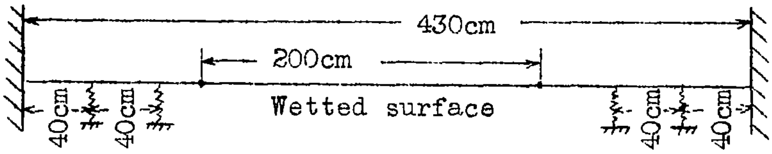

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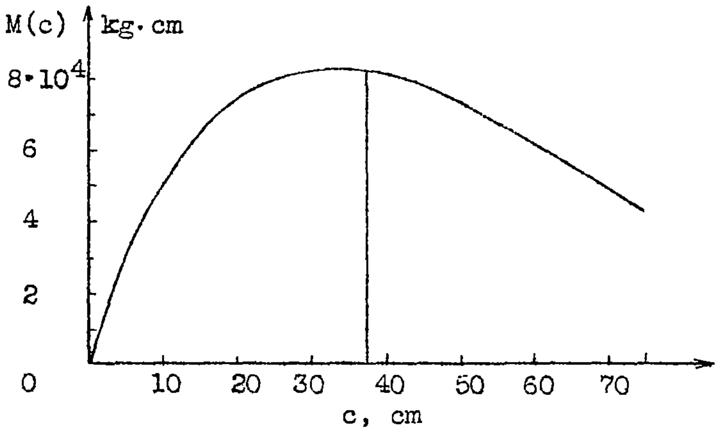

Figure 12.